Least-squares Inversion for Imaging the Full Seismic Wavefield

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Summary

We present a Least-Squares inversion solution for depth migration of the full seismic wavefield including both primaries and high-order reflections. Conventional seismic processing uses only primary reflections for imaging. It treats high-order reflected events as noise and removes them before migration. Separated Wavefield Imaging (SWIM) takes advantage of the extended illumination provided by surface-multiple energy and delivers high-resolution images of the subsurface. Primary and high-order reflections are complementary, and can augment the overall imaging results when properly combined. However, imaging the full wavefield of both primaries and multiples in a standard migration has been challenging for balancing the contribution of each component. In addition, imaging of the full wavefield includes a blending process, which produces crosstalk. Inversion automatically weighs the primary and multiple reflection energy and directly estimates the earth reflectivity that explains boundary data without matching the crosstalk. Successful applications to both synthetic and field data examples demonstrate that Least-Square Full Wavefield Migration (LS-FWM) resolves the crosstalk issue in FWM and greatly improves the imaging illumination and resolution compared to conventional migration.
Introduction

Depth migration of primary reflections often yields an image with insufficient illumination and resolution that can be attributed to limitations in both the acquisition geometry and processing technology employed. Multiple reflections illuminate a much wider range of the model, especially for shallow targets. Separated Wavefield Imaging (SWIM) effectively utilizes all the receivers as virtual sources and exploits the extended illumination provided by surface-multiple energy (Lu et al., 2015).

SWIM and standard migration of primaries are complementary and can augment the overall imaging results when combined properly; however, it is challenging to weigh the contribution of primaries and each multiple component in a standard migration. In addition, a conventional migration of full reflected wavefield (including primaries and surface related multiples) shows strong crosstalk due to the blending nature of the algorithm. Although certain processes such as the deconvolution imaging condition or crosstalk prediction and subtraction can be applied to attenuate crosstalk (Lu et al., 2016), none of them fundamentally resolves the problem.

Linear inversion applications to seismic imaging have been well established (e.g. Schuster, 1993; Nemeth et al., 1999; Prucha and Biondi, 2002; Lu et al., 2018). Least-Squares Migration (LSM) poses depth imaging as a linear inversion problem and solves for an earth reflectivity that minimizes the difference between Born modelled data and the recorded one. Compared to conventional migration of primaries, the least-squares migration of the full wavefield takes advantage of the enhanced illumination from multiples and the power of linear inversion, to produce an image with super resolution and free of crosstalk noise.

In here, we discuss a linear inversion algorithm for imaging the full reflected seismic wavefield including both primaries and surface related multiples. Successful applications to both synthetic and field data examples demonstrate that the inversion solution resolves the crosstalk issue in Full Wavefield Migration (FWM) and improves the illumination and resolution of seismic images compared to conventional migration.

Theory

For observed data $d_{obs}$, seismic imaging aims at solving for the earth reflectivity. The standard depth migration produces an approximation to the reflectivity, $m$

$$m = L^* d_{obs},$$

where operator $L^*$ is the adjoint of the modeling operator $L$. This algorithm is able to produce a structural image of the earth; however, the result often displays uneven illumination, limited bandwidth and wavenumber content, because the migration is not the inverse of a modeling operation.

Least-squares migration (LSM) resolves the inverse of the modeling process

$$m = (L^* L)^{-1} L^* d_{obs},$$

by solving a minimization problem

$$m = \arg \min_m \frac{1}{2} \| d_{obs} - Lm \|^2_2.$$

Generally, two distinct methods can be employed to estimate the model, $m$: the first, explicitly computes the matrix $L^* L$ and its inverse, or alternatively, an implicit approach iteratively inverts the
operator $L$. Here, we discuss the iterative LSM algorithm, which is implemented in a migration/demigration framework (Lu et al., 2018).

Inversion solutions for conventional primary migration and full wavefield migration are differentiated by using different boundary data as summarized in Table 1. In Least-Squares Migration (LSM) of primaries, an impulse source is used as the boundary condition for the modelling operator $L$, and the up-going primary wavefield $P_{up}$ (primaries) acts as the boundary observation $d_{obs}$ for inversion. In Least-Squares Full Wavefield Migration (LS-FWM), the down-going wavefield $P_{down}$ is combined with an impulsive source wavefield and used as the boundary condition for the modelling operator $L$. The total up-going wavefield $P_{up}$ (total) consisting of both primaries and multiples are the boundary observation $d_{obs}$ for inversion.

<table>
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<tr>
<th>Boundary condition for $L$</th>
<th>Boundary observation $d_{obs}$</th>
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<td><strong>LSM</strong></td>
<td>impulsive source</td>
</tr>
<tr>
<td><strong>LS-FWM</strong></td>
<td>impulsive source + $P_{down}$</td>
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*Table 1: Boundary conditions of Least-Squares solutions for migration of primaries (LSM) and Full Wavefield Migration (LS-FWM).*

Since FWM images both primary and surface related multiples simultaneously, it can produce improved images in comparison to standard migration of primaries only. However, it is challenging to balance the contribution of primary and multiple components during the migration procedure. In addition, the FWM results are contaminated by interfering events due to the blending process. Consequently, crosstalk noise is generated by the interference terms in the encoded migration algorithm. On the other hand, the least-squares solution solves an inverse problem that excludes the cross terms [Equation 2]. LS-FWM computes a reflectivity image that explains the observed data without involving the interference noise, and automatically weights the energy of primaries and each multiple component.

**Examples**

We use the 2D synthetic Sigsbee2b data (Figure 1) to demonstrate the power of LS-FWM in attenuating the crosstalk. In Figure 1A, strong crosstalk noise is observed in the lower part of the section (indicated by arrows), it is successfully mitigated in the LS-FWM result in Figure 1B. In addition, LS-FWM significantly improves the imaging illumination and resolution beneath the salt body.

Application of LS-FWM to a 3D WAZ field data from the Gulf of Mexico demonstrates similar benefits from the inversion solution. A depth slice from LS-FWM (Figure 2) shows higher resolution compared to that of FWM. Similar uplift in the image is observed when comparing vertical sections. Besides the improvement in resolution, the crosstalk noise is attenuated successfully by LS-FWM.

We also applied LS-FWM to an Ocean Bottom Cable (OBC) data from the North Sea. The LS-FWM results are compared to the ones from FWM using either a cross-correlation or a deconvolution imaging condition (Figure 3). The FWM image displays strong reverberation interferences and coherent crosstalk (indicated by red arrows). The deconvolution imaging condition (Lu et al., 2015) can mitigate the short period reverberations; however, the longer period coherent crosstalk still persists (Figure 3E). LS-FWM further reduces the residual crosstalk in the deconvolution FWM result and uncovers the structures beneath the noise (Figures 3F). LS-FWM also improves the temporal and spatial resolution and produces the overall best image.
Figure 1: FWM (A) vs LS-FWM (B) with synthetic 2D Sigsbee2b data. LS-FWM resolves the crosstalk (red arrows) issue and improves imaging illumination.

Figure 2: FWM and LS-FWM images from a 3D WAZ Gulf of Mexico dataset: depth slices are at 1150m. LS-FWM mitigates the crosstalk in FWM, and improves imaging illumination and resolution.
Conclusions

We present a Least-Squares inversion algorithm for Full Wavefield Migration (LS-FWM). Successful applications to both synthetic and field data examples demonstrate that LS-FWM greatly improves the illumination and resolution of seismic images. LS-FWM resolves the crosstalk problem in SWIM and FWM and effectively images of the full reflected wavefield, including primaries and multiples.

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References