Seismic Data Up-sampling beyond Aliasing Using Polynomial Phase Signals

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SUMMARY

This paper proposes a new trace up-sampling method for seismic data. The method is based on building a local time-space model of the seismic data following some physical assumptions about its structure. The model simplifies to a superposition of second order polynomial phase signals, which are estimated using a matching pursuit algorithm. The estimated model is then used to generate traces at the desired locations. The proposed method performs up-sampling beyond aliasing thanks to the ability of its matching pursuit basis to model aliasing. It differs from Fourier-based matching methods in the inclusion of a curvature term in the matching basis. The method does not rely on priors from un-aliased part of the data to build the aliasing protection. A good signal to noise ratio is the only condition needed instead. The method is tested on real data and compared to conventional up-sampling solutions. Its performance is remarkably good particularly when it comes to up-sampling highly aliased data.

Introduction

Spatial aliasing causes many problems during the processing of seismic data. For example, transform based processes such as Radon de-multiple will suffer from artifacts when the input data are spatially aliased. In the migration process, the aliased energy will not be moved to the right position and instead it will scatter as quasi-coherent noise in the final image. Performing trace up-sampling beyond aliasing is therefore important and it is required at different stages in a typical seismic data processing sequence.

Standard techniques for regular trace interpolation beyond aliasing include Spitz's frequency-space ($f-x$) interpolator using prediction error filtering (Spitz, 1991) and the frequency-wavenumber ($f-k$) domain spectrum un-wrapping (Gulunay and Chambers, 1997). The $f-x$ prediction method is based on deriving the interpolation model for a given temporal frequency f from the data at a non-aliased frequency, usually at $f/2$. Despite the originality of this technique, the quality of its interpolation depends on having a good signal to noise ratio (SNR) for the non-aliased part of the data. This interpolation fails when the data are highly aliased. The high degree of aliasing pushes down the frequency used to build the interpolating model to the low end of the spectrum where physically the SNR tends to be small. Unwrapping the $f-k$ spectrum is based on deriving an interpolation operator in the $f-k$ domain that relates interleaved traces in the input data. This method is less sensitive to the noise level in the data, but likewise fails when the data are highly aliased. It also performs less effectively for conflicting dips and complex geologies.

This abstract presents a new method for up-sampling of seismic data. It is based on a local decomposition of the seismic data into a superposition of events with polynomial move-out. The solution can be formulated as a data approximation problem using a dictionary of polynomial phase signals (PPS) in the frequency-space domain. The parameters of each PPS are estimated from the input data using a matching pursuit (MP) algorithm. The proposed method is different from the Fourier basis matching interpolation methods (Nguyen and Winnett, 2011) in the inclusion of a second order polynomial phase term in the matching basis to better model the curvature of seismic events.

Method description

Any local window of seismic data comprising N_x equally spaced traces can be modeled as the superposition of K seismic events, where the kth event is defined by a wavelet $w_k(t)$ and a spatially dependent move-out $\tau_k(x)$, i.e.,

$$D(t,x) = \sum_{k=0}^{K-1} w_k\big(t-\tau_k(x)\big) \tag{1}$$

A good approximation of this move-out is a polynomial of order 2, i.e. $\tau_k(x) \approx p_k x + q_k x^2$. Here p_k and q_k represent respectively the dip and the curvature of the kth event in the data model. In the frequency domain Eq. (1) can be written as

$$D(f,x) = \sum_{k=0}^{K-1} W_k(f)\exp\big(j2\pi f(p_k x + q_k x^2)\big), \tag{2}$$

Eq. (2) models the data in the $f-x$ domain as a superposition of K polynomial phase signals (PPS)(Peleg and Friedlander, 1995). The interest in PPS is motivated by a number of applications in the field of radar and radio communication where the processed signals are considered to be non-stationary. An estimate of the parameters $\Theta_K = \{\hat{W}_k(f), \hat{p}_k, \hat{q}_k\}_{k=0}^{K-1}$ is obtained by a matching pursuit (MP) algorithm (Mallat and Zhang, 1993). A pseudo-code of the Up-sampling algorithms using PPS (UPPS) is shown below

Data examples

To evaluate the quality of the up-sampling using UPPS, a shot record data (Figure 1(a)) is decimated by a factor of 2 and odd-number traces are used to generate the even-number traces. Figure 1(c-d) shows

Algorithm 1 UPPS algorithm. R_k : residual, Q_k : up-sampled data at the kth iteration,$\bar{x}$ up-sampling locations

FFT$\big(D(t,x)\big) \rightarrow D(f,x)$, initialise: $R_0(f,x) = D(f,x)$, $Q_0(f,\bar{x}) = \mathbf{0}$

for $k = 1$ to K_{max} **do**

    Find $\quad (\hat{p}_k, \hat{q}_k) = \arg\max_{(p,q)} \sum_f \left| \sum_{n=0}^{N_x-1} R_{k-1}(f,x) \exp\big(-j2\pi f(px+qx^2)\big) \right|^2$

    Compute $\hat{W}_k(f) = \frac{1}{N_x} \sum_{n=0}^{N_x-1} R_{k-1}(f,x) \exp\big(-j2\pi f(\hat{p}_k x + \hat{q}_k x^2)\big)$

    Update $R_k(f,x) = R_{k-1}(f,x) - \hat{W}_k(f) \exp\big(j2\pi f(\hat{p}_k x + \hat{q}_k x^2)\big)$

        $Q_k(f,\bar{x}) = Q_{k-1}(f,\bar{x}) + \hat{W}_k(f) \exp\big(j2\pi f(\hat{p}_k \bar{x}_n + \hat{q}_k \bar{x}_n^2)\big)$

    Compute $E(k)^2 = \sum_f \sum_{n=0}^{N_x-1} |R_k(f,x)|^2 / \sum_f \sum_{n=0}^{N_x-1} |D(f,x)|^2$

    if $E(k) < \varepsilon$ **break**

end for

IFFT$\big(Q_k(f,\bar{x})\big) \rightarrow Q_k(t,\bar{x})$

the target traces that we want to regenerate. The input to UPPS suffers from double aliasing that starts at about 30 Hz. The reconstructed traces are shown in Figure 1(e-f). The difference section (true - reconstructed) scaled-up by a factor of 10 is shown in Figure 1(j-k). Some approximation errors are observed in the water bottom, the shallow reflections below it and also in the refractions recorded at far offsets. The rest of the difference is dominated by random incoherent noise. Most of the reconstruction error energy is in the aliased part of the spectrum and also below 12 Hz where the residual swell noise resides. In the signal cone and up to 60 Hz, the reconstruction error was the smallest. This initial assessment reveals that (i) the quality of interpolation is excellent for un-aliased events with good SNR levels and (ii) interpolation errors for aliased events have comparable energy level to the background noise.

To highlight the effect of including the curvature, we consider the local data window shown in Figure 2(a). The data consist of 24 traces which are taken from a common near-offset channel section around the water bottom. The left leg of the diffractions are visible, yet not with a good resolution. However the right leg is very weak (showed by arrows in Figure 2(a)). The result of a 3:1 up-sampling using a Fourier based matching pursuit interpolation (Nguyen and Winnett, 2011) is shown in Figure 2(b). The

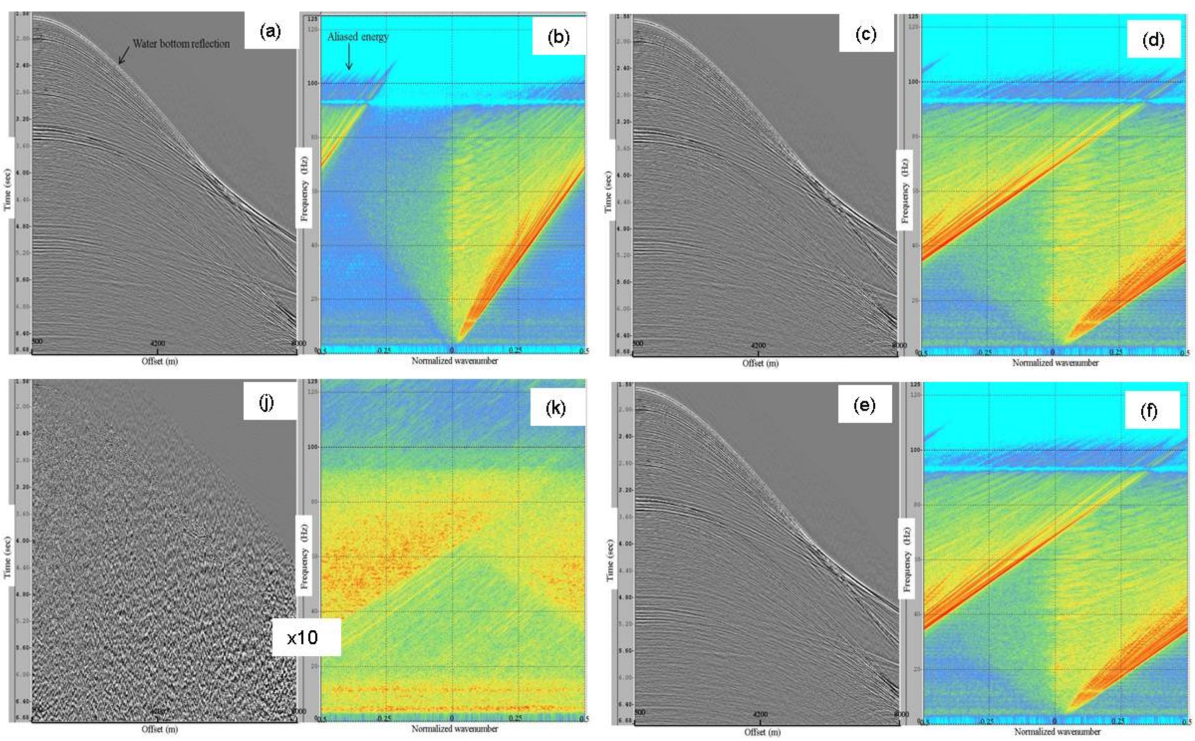

Figure 1: Up-sampling interleaved traces from shot record (a) with its $f-k$ spectrum. True data (c-d), reconstructed data (e-f) and difference scaled by a factor of 10 (j-k). Data Courtesy BG group

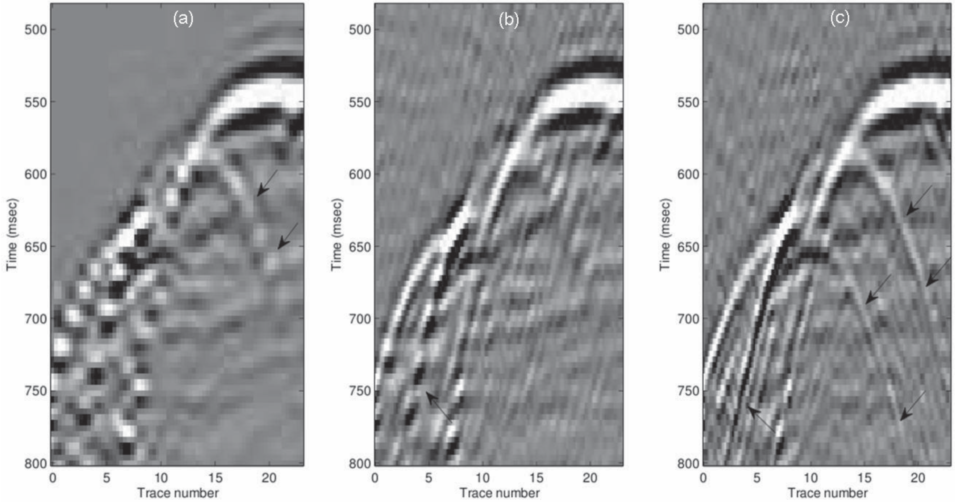

Figure 2: Local window of 24 traces taken from a common channel section around the water bottom (a). The results of 3:1 up-sampling using Fourier based matching pursuit (b) and UPPS with curvature term (c)

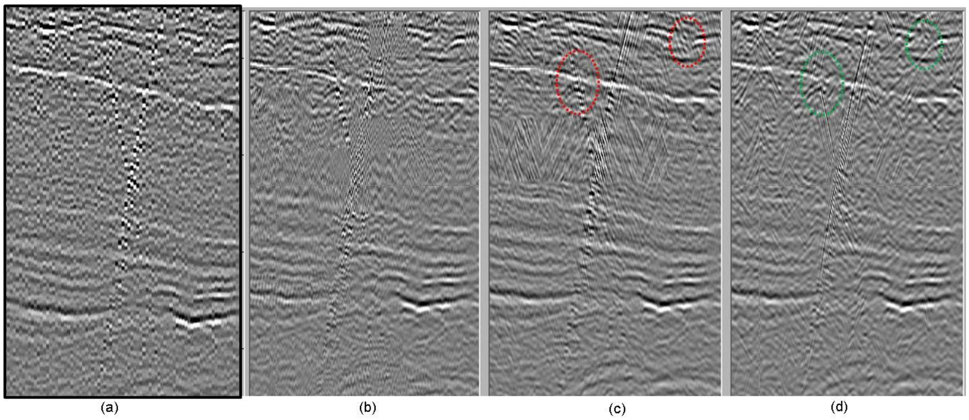

Figure 3: Section from a common channel profile (a) and the results of 4:1 up-sampling using Spitz's method (b), Gulunay (c) and UPPS (d). Data Courtesy BG group

resolution of the left leg of the diffractions has improved, particularly when they are strong. On the other hand, the right leg of the diffractions have not been properly interpolated. The result of the same up-sampling with the curvature term included using UPPS is shown in Figure 2(c). One clearly can see in the improvement in the resolution across all events. Finally UPPS is compared respectively with Spitz $f-x$ and Gulunay $f-k$ interpolation methods, which both are two conventional methods currently used in practice. Figure 3 shows a zoomed part of an input common channel section and the result of a 4:1 up-sampling using each method. Compared with the conventional methods, UPPS achieves a good up-sampling in case of conflicting dips, particularly the (encircled) weaker energy event was not distorted after interpolation. The visible jittery noise in the interpolation obtained by Spitz's methods is common in cases where the lower frequencies in the data have a weak SNR. UPPS produced the best interpolation for the steeply dipping, highly aliased diffraction event. Figure 4 shows the $f-k$ spectra of a window around the dipping diffraction for the input and the output of the different up-sampling methods. The diffraction before interpolation, shown in Figure 4(a), is highly aliased and its energy is scattered in the $f-k$ plane. This represents a challenging case for any interpolation methodology. The result of Spitz's method shown in Figure 4(b) indicates that the aliased energy is mapped around the zero and the Nyquist wavenumber, hence failing completely to un-wrapp the spectrum and leading to a noisy and distorted output. Gulunay's method shown in Figure 4(c) put the aliased event in a wrong position therefore producing an aliased output. On the other hand, UPPS shown in Figure 4(d) maps properly the aliased event in the $f-k$ spectrum and results in a correct up-sampling.

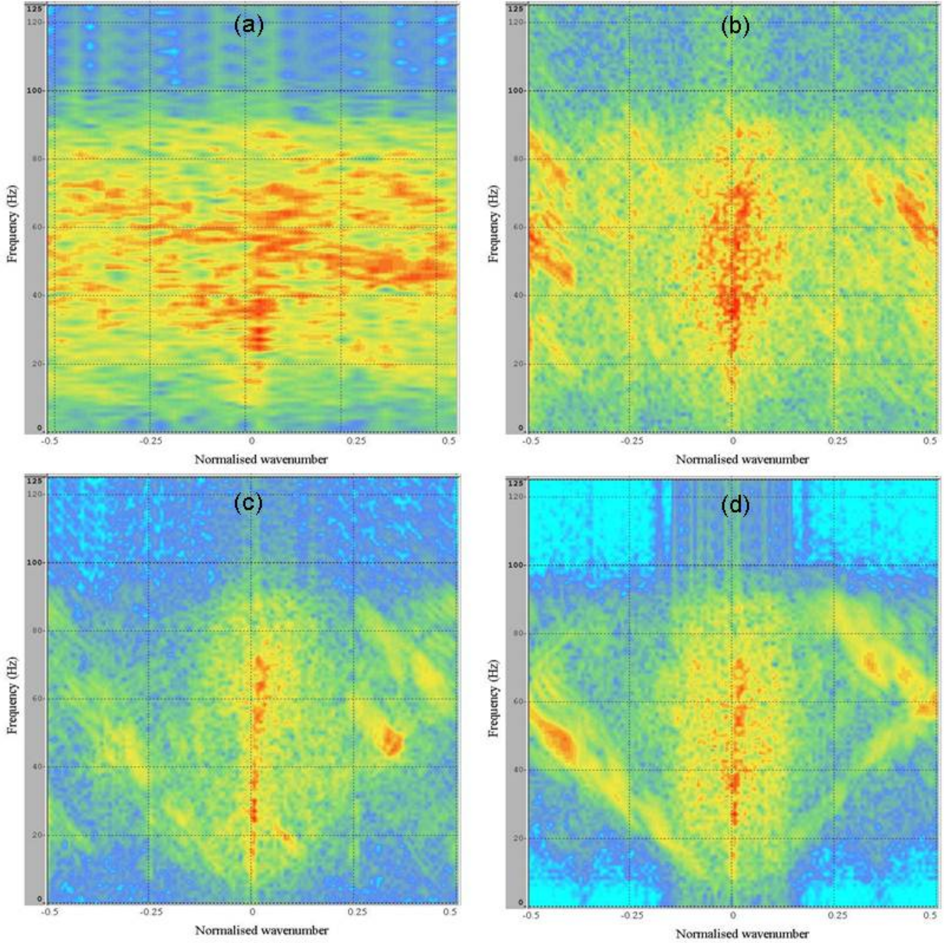

Figure 4: Frequency-wavenumber spectrum of the input data (a), and 4:1 up-sampling by Spitz's (b), Gulunay (c) and UPPS (d)

Conclusion

This paper proposes a new trace up-sampling method for seismic data that belongs to the family of matching pursuit technologies. Its main strength resides in its ability to interpolate highly aliased data and does not need to build the anti-aliasing protection from the low frequencies. This is particularly useful when the data exhibits weak SNR for low frequencies.

Acknowledgement

The authors would like to thank BG group for the permission to show the data and Petroleum GeoServices (PGS) for supporting this work.

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