SUMMARY

Efficient removal of the ghosts is key to achieving broadband seismic data. Accurate deghosting on the receiver-side can only be performed when both pressure and vertical particle velocity information is available. We use acoustic reciprocity to derive a relationship that defines the total pressure wavefield as a function of the up-going pressure wavefield and the ghost function. Utilizing this relationship, and assuming the shape of the sea surface is known, we propose a pre-stack deghosting method based on integral inversion for a pressure-only dataset. Deghosting by spectral division with a flat sea surface or a statistical ghost function is shown to be special cases of this new inversion based method. The behavior of the pressure ghost function under rough sea condition is analysed in comparison to the flat and the statistical pressure ghost functions. The error of deghosting rough sea pressure-only data with a flat sea surface or a statistical ghost function is quantified using both synthetic and real seismic data.
Introduction

Broadband seismic data contains a wealth of information about the subsurface. The broader frequency range in the data allows imaging with higher resolution and provides information about deeper targets. Conventional pressure-only data contain source-side and receiver-side ghost events that reduce noticeably the usable bandwidth of the data. Removing receiver-side ghost events requires both pressure and vertical particle velocity measurements (Day et al., 2013). When the only information available is the pressure measurement, deghosting is performed by making an assumption about the sea surface (Amundsen et al., 1995). In this paper, we focus on the receiver-side ghost and analyse the behaviour of the ghost under rough sea conditions. We first derive a theoretical deghosting method for pressure-only measurements based on integral inversion with a priori knowledge about the sea surface shape. In practice, the actual time varying sea surface shape is not known, so the purpose of this work is to be able to study and quantify the effect of rough sea surface on deghosting using pressure-only data. The formulations with a flat sea surface assumption and using a statistical reflection coefficient are special cases of the integral inversion method. Finally, comparisons of the three methods on synthetic and real seismic data are shown.

Method

We consider a marine seismic data acquisition configuration with sources at $r_s = (x_s, z_s)$ and receivers at $r_r = (x_r, z_r < z_s)$. Utilizing acoustic reciprocity of the convolution type (Fokkema and van den Berg, 1993), the total pressure wavefield $P_{tot}$ can be constructed as a result of coupling two states A and B. In state A, we generate a pressure field at the source locations and measure the up-going wavefield $P_{upA}$ at an arbitrary separation level $r_{sep} = (x_{sep}, z_{sep} < z_s)$. In state B, we generate a pressure field at the receiver locations with virtual sources shaped like a Dirac-delta pulse and measure the down-going wavefield $P_{dnB}$ at the separation level.

Formally, the coupling process can be written in the frequency-wavenumber domain as

$$P_{tot}(\omega, k_r, z_r | k_s, z_s) = \frac{1}{(2\pi)^2} \int_{-\infty}^{\infty} -2i k_z P_{dnB}(\omega, -k_z, z_{sep} | k_r, z_r) P_{upA}(\omega, k_z, z_{sep} | k_s, z_s) dk_z,$$  \hspace{1cm} (1)

where $k_r, k_z$ and $k_{sep}$ are wavenumber vectors, respectively, at the receivers, sources, and separation levels; $\omega$ is the angular frequency and $k_z$ is the vertical wavenumber. The down-going pressure wavefield in state B is related to the pressure ghost function $G_p$ by

$$G_p(\omega, k_{sep}, z_{sep} | k_r, z_r) = -2i k_z P_{dnB}(\omega, k_z, z_{sep} | k_r, z_r).$$  \hspace{1cm} (2)

Replacing the infinite integration in Eq. (1) by a finite discrete summation, inserting Eq. (2) and considering a single frequency, Eq. (1) can be written in matrix form as

$$P_{tot} = \frac{dk_{sep}}{(2\pi)^2} G_p P_{upA}.$$

If the shape of the sea-surface is precisely known, we can model $G_p$ (Asgedom et al., 2014) and obtain $P_{upA}$ by removing the effect of the ghost from $P_{tot}$ using integral inversion. The integral inversion based deghosting technique can only be performed in the common-shot domain or for all sources and receivers used to acquire the data. It is also worth noting that the ghost function $G_p$ can have deep notches causing instabilities in the inversion.

When there is no information available about the sea surface, we often assume it is flat with a reflection coefficient of -1. Utilizing the analytic form of the Green’s function in a homogenous medium with a free surface boundary condition, and considering translational shift invariance of $G_p$ with $z_{sep} = z_r$, the matrix equation in Eq. (3) reduces to spectral multiplication given by

$$P_{tot}(\omega, k_r, z_r | x_s = 0, z_s) = G_p^{lat}(\omega, k_r, z_r | x_s = 0, z_r) P_{upA}(\omega, k_r, z_r | x_s = 0, z_s),$$  \hspace{1cm} (4)
where \( G_p^{\text{flat}}(\omega, k_r, z_r | x_s = 0, z_r) = 1 - \exp(-2ik_z z_r) \) is the flat sea surface ghost function. Here, deghosting can be performed by spectrally dividing the total pressure wavefield in the common-shot domain with the flat sea surface ghost function. Another approach for deghosting marine seismic data acquired in rough seas relies on the statistical behavior of the sea surface. Assuming the sea surface height follows a Gaussian distribution with zero mean and unit variance, the statistical reflection coefficient for the coherently scattered part of the wavefield is given by (Ogilvy, 1987)

\[
R_{\text{stat}}(\omega, k_r) = \exp(-2(k_z \sigma)^2),
\]  

where \( \sigma \) is the Root Mean Square (RMS) height of the sea surface. We can now use \( R_{\text{stat}} \) to construct a statistical pressure ghost function given by

\[
G_p^{\text{stat}}(\omega, k_r, z_r | x_s = 0, z_r) = 1 - R_{\text{stat}}(\omega, k_r) \exp(-2ik_z z_r).
\]

Both the flat sea surface and statistical ghost functions can be viewed as a filter which transforms the up-going pressure wavefield into the total pressure wavefield. Following the same principle, we can define the true pressure ghost filter \( G_p^{\text{filter}} \) for any sea surface condition as

\[
G_p^{\text{filter}}(\omega, k_r) = \frac{P_{\text{tot}}(\omega, k_r)}{P_{\text{up}}(\omega, k_r)}.
\]

Note that the true pressure ghost filter can only be determined if we have both the total and the up-going pressure wavefields.

**Synthetic Data Examples**

Two dimensional synthetic data was generated from a homogenous half-space model bounded by a rough sea surface with a significant wave height of 4.6m. The source and receivers are located respectively at a depth of 50m and 15m. The separation level was chosen to be 5m below the receivers. Figures 1a - 1d show the modelled total and up-going pressure wavefields both in time-space and frequency-wavenumber (FK) domains. The effect of the rough sea surface introduces fluctuations in the total pressure wavefield (cf. Fig. 1a compare to Fig. 1b) which shows up as a diversity of notch structures in the spectrum (cf. Fig. 1c). The up-going (i.e. ghost free) pressure wavefield is characterized both by smooth amplitudes in time-space (cf. Fig. 1b) and smooth spectrum in FK (cf. Fig. 1d).

![Figure 1](image)

**Figure 1** Modelled total (a) and up-going (b) pressure in time-space. The corresponding amplitude spectra are shown respectively for the total and up-going pressure in (c) and (d).

The true pressure ghost filter, computed from Eq. (7) using the modelled total and up-going pressure wavefields, is used as a reference and is compared with the flat sea surface and statistical pressure ghost functions. The amplitude spectrum of the true pressure ghost filter is shown in Fig. 2a. Note the diversity of notch structures and the variation in amplitude due to the presence of incoherently scattered energy from the rough sea surface. The amplitude spectra of the flat sea surface and statistical pressure ghost functions, computed using Eqs. (4) and (6), are shown in Figs. 2b and 2c, respectively. The flat sea surface ghost function has very deep notches for all frequencies while the notch depth of the statistical ghost function decreases with increasing frequency. However, low frequency notches remain deep for the statistical ghost function as well. Both the statistical and the flat sea surface ghost functions cannot predict the incoherently scattered energy observed in the true ghost filter.
Assuming we know the shape of the sea surface, deghosting was performed based on inversion using Eq. (3). The rough sea surface ghost function, required to perform the inversion, was modelled using the known shape of the sea surface. The deghosting result in Fig. 3a shows an excellent match with the modelled up-going wavefield in Fig. 1b. Deghosting results by spectral division are shown in Fig. 3b using the flat sea surface ghost function and in Fig. 3c using the statistical ghost function. As expected, the result of flat sea surface deghosting becomes unstable at the ghost notch locations. Deghosting with the statistical ghost function shows a residual down-going wavefield remaining in the deghosted result. Both the flat sea surface and statistical deghosting methods have significantly smaller errors towards the lower frequencies.

In order to quantify the error of each deghosting methods, we computed the relative error between the modelled up-going wavefield and the deghosting results. The relative error of the inversion result shown in Fig. 3d is very small and concentrated around the pressure ghost notches. The relative errors of both the flat sea surface (cf. Fig. 3e) and statistical (cf. Fig. 3f) deghosting results show significant errors due to not accounting for incoherent scattering. These errors increase with frequency. The flat sea deghosting results exhibit strong singularities around the notches which can also be seen in the statistical deghosting result, although decreasing with frequency.

Field Data Example

Field data acquired offshore the Falkland Islands was used to compare the behaviour of the true ghost filter with the flat sea surface and the statistical ghost functions. The data was acquired with collocated pressure and vertical particle velocity sensors which enabled a correct separation of the
total pressure wavefield into its up-going and down-going parts. The up-going pressure wavefield in FK is shown in Fig. 4a. The true ghost filter can now be generated by performing spectral division between the total and the up-going pressure wavefields. Figure 4b shows the derived ghost filter in FK for the selected shot gather. Figures 4c and 4d show the amplitude spectra in FK of the flat sea surface and statistical ghost functions, respectively. To compute the statistical ghost function, an estimated RMS of the sea surface height (~0.65m) was obtained from imaging the sea surface variation (Orji et al., 2013). Note that the second pressure ghost notch is deep in all three instances and this can create instabilities in the deghosting process. Note the similarity in the notch diversity and amplitude variation, as a result of incoherent scattering, between the ghost filter derived from the data shown in 4b, and the calculated true ghost filter from the synthetic data example (cf. Fig. 2a). These effects cannot properly be accounted for when using the flat sea surface or statistical ghost function.

Figure 4 Amplitude spectra of the up-going pressure wavefield (a) and its true pressure ghost filter (b). The ghost functions assuming flat sea surface (c) and using statistical reflection coefficient (d).

Conclusions

From wave theory, correct receiver-side deghosting can only be performed when both the pressure and the vertical particle velocity information are available. If the sea surface shape is known, the inversion based method proposed in this paper could be used to accurately remove the ghost from the data. In reality, determining the time varying sea surface by independent means is not practical. Therefore dual-sensor measurements are needed to be able to do correct receiver-side deghosting. When only statistical information on the wave heights is available, the statistical deghosting method may be used but instabilities at the lower frequency notches have to be handled and incoherent scattering is not taken into account. The flat sea surface deghosting method will require similar stabilization at notches also at higher frequencies. Note that in the low frequency range, all three methods converge to the correct solution for up-going pressure field. The findings in this work can be used as basis to estimate the errors that are made by assuming a flat sea surface as a function of the actual sea state.

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References


